Erratum: Symplectic particle-in-cell model for space-charge beam dynamics simulation [Phys. Rev. Accel. Beams 21, 054201 (2018)]

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There is a typographical error in Eq. (16) of the paper. The correct equation is

$$H_2 = \frac{K}{4}\sum_{i=1}^{N_p}\sum_{j=1}^{N_p}\varphi(\mathbf{r}_i, \mathbf{r}_j) \tag{1}$$

This affects the following Eqs. (32), (34)–(37), (39)–(41), and (49). The correct Eq. (32) is

$$\begin{aligned} H_2 = 4\pi \frac{K}{4}\frac{4}{ab}\frac{1}{N_p}\sum_{i=1}^{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\frac{1}{\Delta x\Delta y}\int_0^a\int_0^b S(x-x_j)S(y-y_j)\sin(\alpha_l x)\sin(\beta_m y)dxdy \\ \times \frac{1}{\Delta x\Delta y}\int_0^a\int_0^b S(x-x_i)S(y-y_i)\sin(\alpha_l x)\sin(\beta_m y)dxdy. \end{aligned} \tag{2}$$

The correct Eq. (34) is

$$H_2 = 4\pi \frac{K}{4}\frac{4}{ab}\frac{1}{N_p}\sum_{i=1}^{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\sin(\alpha_l x_j)\sin(\beta_m y_j)\sin(\alpha_l x_i)\sin(\beta_m y_i). \tag{3}$$

The one-step symplectic transfer map $\mathcal{M}_2$ of the particle i with this Hamiltonian is given as

$$\begin{aligned} p_{xi}(\tau) &= p_{xi}(0) - \tau 4\pi \frac{K}{2}\frac{4}{ab}\frac{1}{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{\alpha_l}{\gamma_{lm}^2}\sin(\alpha_l x_j)\sin(\beta_m y_j)\cos(\alpha_l x_i)\sin(\beta_m y_i), \\ p_{yi}(\tau) &= p_{yi}(0) - \tau 4\pi \frac{K}{2}\frac{4}{ab}\frac{1}{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{\beta_m}{\gamma_{lm}^2}\sin(\alpha_l x_j)\sin(\beta_m y_j)\sin(\alpha_l x_i)\cos(\beta_m y_i). \end{aligned} \tag{4}$$

The correct Eqs. (36)–(37) are

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$$p_{xi}(\tau) = p_{xi}(0) - \tau 4\pi \frac{K}{2}\frac{4}{ab}\frac{1}{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\frac{1}{\Delta x\Delta y}\int_0^a\int_0^b S(x-x_j)S(y-y_j)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$
$$\times \frac{1}{\Delta x\Delta y}\int_0^a\int_0^b \frac{\partial S(x-x_i)}{\partial x_i}S(y-y_i)\sin(\alpha_l x)\sin(\beta_m y)dxdy,$$
$$p_{yi}(\tau) = p_{yi}(0) - \tau 4\pi \frac{K}{2}\frac{4}{ab}\frac{1}{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\frac{1}{\Delta x\Delta y}\int_0^a\int_0^b S(x-x_j)S(y-y_j)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$
$$\times \frac{1}{\Delta x\Delta y}\int_0^a\int_0^b S(x-x_i)\frac{\partial S(y-y_i)}{\partial y_i}\sin(\alpha_l x)\sin(\beta_m y)dxdy, \quad (5)$$

$$p_{xi}(\tau) = p_{xi}(0) - \tau 4\pi \frac{K}{2}\frac{4}{ab}\frac{1}{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\sum_{I'}\sum_{J'}S(x_{I'}-x_j)S(y_{J'}-y_j)\sin(\alpha_l x_{I'})\sin(\beta_m y_{J'})$$
$$\times \sum_I\sum_J \frac{\partial S(x_I-x_i)}{\partial x_i}S(y_J-y_i)\sin(\alpha_l x_I)\sin(\beta_m y_J),$$
$$p_{yi}(\tau) = p_{yi}(0) - \tau 4\pi \frac{K}{2}\frac{4}{ab}\frac{1}{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\sum_{I'}\sum_{J'}S(x_{I'}-x_j)S(y_{J'}-y_j)\sin(\alpha_l x_{I'})\sin(\beta_m y_{J'})$$
$$\times \sum_I\sum_J S(x_I-x_i)\frac{\partial S(y_I-y_i)}{\partial y_i}\sin(\alpha_l x_I)\sin(\beta_m y_J). \quad (6)$$

The correct Eqs. (39) and (41) are

$$p_{xi}(\tau) = p_{xi}(0) - \tau 4\pi \frac{K}{2}\sum_I\sum_J \frac{\partial S(x_I-x_i)}{\partial x_i}S(y_J-y_i)\left[\frac{4}{ab}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\right.$$
$$\left.\times \sum_{I'}\sum_{J'}\bar{\rho}(x_{I'},y_{J'})\sin(\alpha_l x_{I'})\sin(\beta_m y_{J'})\sin(\alpha_l x_I)\sin(\beta_m y_J)\right],$$
$$p_{yi}(\tau) = p_{yi}(0) - \tau 4\pi \frac{K}{2}\sum_I\sum_J S(x_I-x_i)\frac{\partial S(y_I-y_i)}{\partial y_i}\left[\frac{4}{ab}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\right.$$
$$\left.\times \sum_{I'}\sum_{J'}\bar{\rho}(x_{I'},y_{J'})\sin(\alpha_l x_{I'})\sin(\beta_m y_{J'})\sin(\alpha_l x_I)\sin(\beta_m y_J)\right], \quad (7)$$

$$p_{xi}(\tau) = p_{xi}(0) - \tau 4\pi \frac{K}{2}\sum_I\sum_J \frac{\partial S(x_I-x_i)}{\partial x_i}S(y_J-y_i)\phi(x_I,y_J),$$
$$p_{yi}(\tau) = p_{yi}(0) - \tau 4\pi \frac{K}{2}\sum_I\sum_J S(x_I-x_i)\frac{\partial S(y_J-y_i)}{\partial y_i}\phi(x_I,y_J). \quad (8)$$

The correct Eq. (49) is

$$p_{xi}(\tau) = p_{xi}(0) + \tau\left(\frac{qE_x^{\mathrm{ext}}}{v_0} - qB_y^{\mathrm{ext}}\right) + \tau 4\pi \frac{K}{2}\sum_I\sum_J S(x_I-x_i)S(y_J-y_i)E_x(x_I,y_J),$$
$$p_{yi}(\tau) = p_{yi}(0) + \tau\left(\frac{qE_y^{\mathrm{ext}}}{v_0} + qB_x^{\mathrm{ext}}\right) + \tau 4\pi \frac{K}{2}\sum_I\sum_J S(x_I-x_i)S(y_J-y_i)E_y(x_I,y_J). \quad (9)$$

The simulation results presented in the original paper were obtained with the correct equations. This modification does not have any effects on the conclusions of the paper.